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Cosmology of antisymmetric tensor field in D-brane universe

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Abstract: We analyze homogeneous, anisotropic cosmology driven by a self-interacting "massive" antisymmetric tensor field $B_{\mu\nu}$ which is present in string theories with D-branes. Time-dependent magnetic B field existing in the early universe can lead to the Bianchi type I universe. Evolutions of such a tensor field are solved exactly or numerically in the universe dominated by vacuum energy, radiation, and B field itself. The matter-like behavior of the B field (dubbed as "B-matter") ensures that the anisotropy disappears at late time and thus becomes unobservable in a reasonable cosmological scenario. Such a feature should be contrasted to the cosmology of the conventional massless antisymmetric tensor field.

Keywords: D-branes, Cosmology of Theories beyond the SM.

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1. Introduction

The antisymmetric tensor field (Kalb-Ramond field) $B_{\mu\nu}$ is an indispensable field in effective theories derived from the string theory. It was considered as a massless degree of freedom in the bulk spacetime and thus it is dual to a pseudo-scalar field h: $H_{\mu\nu\rho} = \partial_{[\mu}B_{\nu\rho]} = \epsilon_{\mu\nu\rho\sigma}\partial^\sigma h$ in four dimensions [1]. Once we take into account D-brane which easily forms a bound state of Dp-brane-fundamental strings [2], such a B field alone becomes gauge non-invariant on the Dp-brane and gauge invariance is restored through its coupling to a U(1) vector field A_μ living on a Dp-brane, i.e. the gauge-invariant field strength is

$$B_{\mu\nu} + 2\pi\alpha' F_{\mu\nu} \equiv \mathcal{B}_{\mu\nu}\,, \qquad \text{where} \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,. \tag{1.1}$$

Note that $\mathcal{B}_{\mu\nu}$ is invariant under the tensor gauge transformation

$$\delta B_{\mu\nu} = \partial_\mu\zeta_\nu - \partial_\nu\zeta_\mu\,, \qquad \delta A_\mu = -\frac{\zeta_\mu}{2\pi\alpha'}$$

including the ordinary U(1) gauge transformation $\zeta_\mu = \partial_\mu\lambda$. Intriguingly enough, the effective action of antisymmetric tensor field derived on the D-brane dictates this abelian tensor field of second rank to be massive and self-interacting [3, 2, 4].

Another indispensable field is gravitational field $g_{\mu\nu}$ and, once it also enters into the playground in the string scale, it is natural to deal with cosmological issues due to the existence of the antisymmetric tensor field. In this paper, we consider the cosmology of the antisymmetric tensor field in a D-brane world which is supposed to contain our universe. The cosmological solutions with the massive antisymmetric tensor field $\mathcal{B}$ are quite different from those with the massless one. In the latter case, a homogenous and isotropic universe can be realized for $H_{0ij} = 0$ and $H_{ijk} = f\epsilon_{ijk}$ with some constant $f \propto \partial_t h$ [5]. Another homogeneous solution leading to an anisotropic Bianchi type I universe takes the configuration of $H_{ijk} = 0$ and $H_{0ij} = \epsilon_{ijk} L_k$ with one non-vanishing constant $L_k \propto \partial_k h$ [6]. Such an anisotropy could be in conflict with observation. Indeed, the anisotropy driven by a non-vanishing L_k grows in an expanding universe [6]. In relation with anisotropy in Bianchi type universes, various issues have been discussed [7]. In the brane world where the antisymmetric tensor field becomes massive, time-dependent homogeneous solutions are described by "magnetic" degrees of freedom $\mathcal{B}_{ij} = \mathcal{B}_{ij}(t)$ leading to $H_{ijk} = 0$ and $H_{0ij} = \partial_t \mathcal{B}_{ij}$, which does not allow isotropic cosmology. Assuming the simplest form of anisotropic universe, Bianchi type I, we will analyze the cosmology with the massive magnetic field $\mathcal{B}_{ij}$ in various situations of inflationary, radiation dominated, or $\mathcal{B}$-dominated era. As we will see, the initial anisotropy is damped away and thus a practically isotropic universe is recovered at late time due to the ordinary matter-like behavior of the $\mathcal{B}$ field.

The paper is organized as follows. In section 2, we describe the four-dimensional low-energy effective action of D-brane world from which the mass of the antisymmetric tensor field and its equations of motion coupled to the gravity are derived. In section 3, we find time-dependent homogenous solutions with one magnetic component $\mathcal{B}_{12}(t) \neq 0$ which leads to the Bianchi type I cosmology. Section 4 concludes the paper.

2. Antisymmetric tensor field on D-brane

Suppose that our universe is a part of a Dp-brane. Then its bosonic sector comprises bulk degrees including the graviton $g_{\mu\nu}$, the dilaton Φ, and the antisymmetric tensor field $B_{\mu\nu}$ of rank-two in addition to U(1) gauge field A_μ living on the Dp-brane. Combining $B_{\mu\nu}$ with $F_{\mu\nu}$ as in (1.1), the bosonic action of our system in string frame is given by the sum of the bulk terms in ten dimensions;

$$S_{10} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g}\, e^{-2\Phi} \left[R - 2\Lambda_{10} + 4(\partial\Phi)^2 - \frac{1}{12} H^2 \right], \tag{2.1}$$

and the brane terms in p spatial dimensions;

$$S_{\mathrm{D}p} = -\mu_p \int d^{p+1}x\, e^{-\Phi} \sqrt{-\det(g + \mathcal{B})}, \tag{2.2}$$

where $\kappa_{10}^2 = \frac{1}{2}(2\pi)^7 \alpha'^4$ and $\mu_p^2 = (\pi/\kappa_{10}^2)(4\pi^2\alpha')^{3-p}$ [4]. Recall that α' defines the string scale; $m_{\mathrm{s}} = \alpha'^{-1/2}$. Here the spacetime indices are not explicitly expressed and the field strength H of the antisymmetric tensor field is again $H_{\mu\nu\rho} = \partial_{[\mu} \mathcal{B}_{\nu\rho]}$ because of Bianchi identity of the U(1) gauge field.

To consider the cosmology of our universe, let us compactify six extra-dimensions with a common radius R_{c} and assume the stabilized dilaton leading to a finite string coupling $g_{\mathrm{s}} = e^{\Phi}$. Then, the four-dimensional effective action becomes

$$S_4 = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} \left[R - 2\Lambda - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} - m_B^2 \sqrt{1 + \frac{1}{2} \mathcal{B}_{\mu\nu}\mathcal{B}^{\mu\nu} - \frac{1}{16} \left(\mathcal{B}^*_{\mu\nu} \mathcal{B}^{\mu\nu} \right)^2 } \right], \tag{2.3}$$

where $\mathcal{B}^*_{\mu\nu} = \sqrt{-g}\epsilon_{\mu\nu\alpha\beta}\mathcal{B}^{\alpha\beta}/2$ with $\epsilon_{0123} = 1$. Here the reduced four-dimensional Planck length κ_4 is determined by $1/\kappa_4^2 = R_c^6/g_s^2\kappa_{10}^2$. The effective action (2.3) up to the quadratic terms shows that the antisymmetric tensor field acquires the mass;

$$m_B = \pi^{\frac{1}{4}}\alpha_s^{\frac{p-3}{16}}\left(\frac{m_c}{m_P}\right)^{\frac{15-p}{8}} m_P\,, \tag{2.4}$$

where $\alpha_s = g_s^2/4\pi$, $m_c = 1/R_c$, and $m_P = 1/\kappa_4 = 2.4\times 10^{18}\,\mathrm{GeV}$. As we will see, the massive $\mathcal{B}$ field (named as B-matter) behaves like a massive scalar in some respects in its cosmological evolution, due to which the intrinsic anisotropy set by directional $\mathcal{B}$ field gets diminished eventually.

From the action (2.3), one finds the following equations of motion for the metric $g_{\mu\nu}$ and the antisymmetric tensor field $\mathcal{B}_{\mu\nu}$

$$\nabla^\mu H_{\mu\nu\rho} - m_B^2\frac{\mathcal{B}_{\nu\rho} - \frac{1}{4}\mathcal{B}^*_{\nu\rho}(\mathcal{B}^*\mathcal{B})}{\sqrt{1+\frac{1}{2}\mathcal{B}^2 - \frac{1}{16}(\mathcal{B}^*\mathcal{B})^2}} = 0\,, \tag{2.5}$$

$$\nabla^\mu\left[\frac{\mathcal{B}_{\mu\nu} - \frac{1}{4}\mathcal{B}^*_{\mu\nu}(\mathcal{B}^*\mathcal{B})}{\sqrt{1+\frac{1}{2}\mathcal{B}^2 - \frac{1}{16}(\mathcal{B}^*\mathcal{B})^2}}\right] = 0\,, \tag{2.6}$$

$$G_{\mu\nu} = \Lambda g_{\mu\nu} + \frac{1}{12}\left(3H_{\mu\lambda\rho}H_\nu{}^{\lambda\rho} - \frac{1}{2}g_{\mu\nu}H^2\right) + \frac{m_B^2}{2}\frac{\mathcal{B}_{\mu\lambda}\mathcal{B}_\nu^\lambda - g_{\mu\nu}\left(1+\frac{1}{2}\mathcal{B}^2\right)}{\sqrt{1+\frac{1}{2}\mathcal{B}^2 - \frac{1}{16}(\mathcal{B}^*\mathcal{B})^2}}\,, \tag{2.7}$$

where ∇_μ is the covariant derivative. The equation (2.6) is also required from the consistency of (2.5), as you can easily check by applying ∇^ν to (2.5). Out of six equations in (2.5), one can find that three of them are dynamical equations and the other three are constraints. Therefore, the field $\mathcal{B}_{\mu\nu}$ contains only three degrees of freedom as anticipated from the discussion of (1.1).

3. Time-dependent homogeneous solutions

In this section we investigate evolutions of homogeneous magnetic components of the antisymmetric tensor field $\mathcal{B}_{ij}$ with and without gravity. In the flat spacetime composed of a D-brane we find an oscillating solution. In the B-dominated D-brane universe, isotropy is recovered despite of the initially-assumed anisotropy, whereas it is not without D-brane.

3.1 Flat spacetime: homogeneous magnetic B-matter

We begin this subsection by turning off the gravitational coupling on the D-brane. We now consider homogeneous configuration $\mathcal{B}_{\mu\nu}(t)$ and look for its time evolution in flat spacetime with $g_{\mu\nu} = \eta_{\mu\nu}$. Time component of the equation (2.6) is trivially satisfied and spatial components become

$$\partial_0\left[\frac{\mathbf{E} - \mathbf{B}(\mathbf{E}\cdot\mathbf{B})}{\sqrt{1-\mathbf{E}^2+\mathbf{B}^2-(\mathbf{E}\cdot\mathbf{B})^2}}\right] = 0\,, \tag{3.1}$$

where $\mathcal{B}_{i0} \equiv (\mathbf{E})^i$ and $\mathcal{B}_{ij} \equiv \epsilon_{0ijk}(\mathbf{B})^k$. This equation (3.1) combined with the $0i$-component of (2.5) reads

$$\mathbf{E} = \mathbf{B}(\mathbf{E} \cdot \mathbf{B})\,. \tag{3.2}$$

This allows a non-trivial solution with $\mathbf{E} = 0$ and $\mathbf{B}(t) \neq 0$. Then, the dynamical equation of the $\mathbf{B}$ field is summarized by

$$\partial_0^2 \mathbf{B} = -m_B^2 \frac{\mathbf{B}}{\sqrt{1+\mathbf{B}^2}}\,. \tag{3.3}$$

Integration of the equation (3.3) gives

$$\mathcal{E} = \frac{1}{2m_B^2}\dot{\mathbf{B}}^2 + V_{\rm eff}(\mathbf{B})\,, \tag{3.4}$$

where $\mathcal{E}$ is an integration constant, $V_{\rm eff}(\mathbf{B}) = \sqrt{1+\mathbf{B}^2}$, and overdot is differentiation over time variable t. Let us assume that the direction of the magnetic field is fixed, i.e. $\hat{\mathbf{k}}$ defined by $\mathbf{B}(t) = B(t)\hat{\mathbf{k}}$ is independent of time. Then (3.4) is rewritten by an integral equation

$$m_B(t - t_0) = \pm \int_{B_0}^{B} \frac{dB}{\sqrt{2(\mathcal{E} - \sqrt{1+B^2})}}\,. \tag{3.5}$$

The only pattern of nontrivial solutions is oscillating one with amplitude $\sqrt{\mathcal{E}^2 - 1}$ for $\mathcal{E} > 1$. This oscillating solution with fixed amplitude is natural at classical level in flat spacetime when a homogeneous condensation of magnetic field is given by an initial condition. In expanding universes, the solution is expected to change to that with oscillation and damping due to growing of the spatial scale factor, which means the dilution of B-matter.

3.2 B-dominated universe

We are interested in the cosmological implications of the large scale fluxes of the antisymmetric tensor field, which might have existed in the early universe. Thus we look for cosmological solutions of (2.5)–(2.7) with appropriate initial conditions and examine the effects on the cosmological evolution. First we consider the case that the universe is dominated by the massive tensor field. Extending the above flat spacetime solution, we take into account the homogeneous configurations with an ansatz $\mathcal{B}_{ij} = \mathcal{B}_{ij}(t)$ and $\mathcal{B}_{0i} = 0$. For general non-vanishing $\mathcal{B}_{ij}(t)$, the right-hand side of (2.7) says that the energy-momentum tensor $T_{\mu\nu}$ of antisymmetric tensor field will have non-vanishing off-diagonal components T_{ij}. To consider the simplest form of anisotropic cosmology, we take only single component of $\mathcal{B}_{ij}$ to be nonzero; namely $\mathcal{B}_{12}(t) \equiv B(t)$ and $\mathcal{B}_{23}(t) = \mathcal{B}_{31}(t) = 0$. Even in this case, $T_{11}(t) = T_{22}(t) \neq T_{33}(t)$ in general and thereby the corresponding metric is not isotropic. Therefore, we take the metric to be of Bianchi type I

$$ds^2 = -dt^2 + a_1(t)^2(dx^1)^2 + a_2(t)^2(dx^2)^2 + a_3(t)^2(dx^3)^2\,. \tag{3.6}$$

The energy-momentum tensor of the B field is written in the form

$$\begin{aligned}(T_B)_\mu{}^\nu &= (T_{\rm bulk})_\mu{}^\nu + (T_{\rm brane})_\mu{}^\nu \\ &= \kappa_4^{-2}\Lambda\delta_\mu{}^\nu + {\rm diag}\,[-\rho_B, -\rho_B, -\rho_B, +\rho_B] + {\rm diag}\,[-\rho_b, -\tilde{\rho}_b, -\tilde{\rho}_b, -\rho_b]\,,\end{aligned} \tag{3.7}$$

where $\rho_B = \dot{B}^2/4\kappa_4^2 a_1^2 a_2^2$, $\rho_b = (m_B^2/2\kappa_4^2)\sqrt{1+B^2/a_1^2 a_2^2}$ and $\tilde{\rho}_b = (m_B^2/2\kappa_4^2)/\sqrt{1+B^2/a_1^2 a_2^2}$. Thus, the Einstein equations (2.7) for the tensor-field-dominated case are

$$\frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} + \frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3} + \frac{\dot{a}_3}{a_3}\frac{\dot{a}_1}{a_1} = \kappa_4^2\rho_B + \kappa_4^2\rho_b + \Lambda\,, \tag{3.8}$$

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3} = \kappa_4^2\rho_B + \kappa_4^2\tilde{\rho}_b + \Lambda\,, \tag{3.9}$$

$$\frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_3}{a_3}\frac{\dot{a}_1}{a_1} = \kappa_4^2\rho_B + \kappa_4^2\tilde{\rho}_b + \Lambda\,, \tag{3.10}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} = -\kappa_4^2\rho_B + \kappa_4^2\rho_b + \Lambda\,. \tag{3.11}$$

The equation of motion for $B(t)$ is

$$\ddot{B} + \left(-\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right)\dot{B} + m_B^2\frac{B}{\sqrt{1+B^2/a_1^2 a_2^2}} = 0\,. \tag{3.12}$$

Introducing $\alpha_i = \ln a_i$, we rewrite the Einstein equations (3.8)–(3.11) as

$$\dot{\alpha}_1\dot{\alpha}_2 + \dot{\alpha}_2\dot{\alpha}_3 + \dot{\alpha}_3\dot{\alpha}_1 = \kappa_4^2\rho_B + \kappa_4^2\rho_b + \Lambda\,, \tag{3.13}$$

$$\ddot{\alpha}_1 + \dot{\alpha}_1(\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3) = \kappa_4^2\rho_b + \Lambda\,, \tag{3.14}$$

$$\ddot{\alpha}_2 + \dot{\alpha}_2(\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3) = \kappa_4^2\rho_b + \Lambda\,, \tag{3.15}$$

$$\ddot{\alpha}_3 + \dot{\alpha}_3(\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3) = 2\kappa_4^2\rho_B + \kappa_4^2\tilde{\rho}_b + \Lambda\,. \tag{3.16}$$

Subtraction of (3.15) from (3.14) gives

$$\ddot{\alpha}_1 - \ddot{\alpha}_2 + (\dot{\alpha}_1 - \dot{\alpha}_2)(\dot{\alpha}_1 + \dot{\alpha}_2 + \dot{\alpha}_3) = 0\,. \tag{3.17}$$

A natural solution is $\dot{\alpha}_1 = \dot{\alpha}_2$ which, with the help of scaling of x^1 and x^2 coordinates, leads to the isotropy in x^1x^2-plane, $\alpha_1 = \alpha_2$ or equivalently $a_1 = a_2$.

3.2.1 Massless limit ($m_B = 0$)

To see the effect of the brane on the spacetime dynamics, let us first consider the limit of $m_B = 0$ which corresponds to either the absence of the brane or the limit of vanishing string coupling g_s, namely the usual massless antisymmetric tensor field. In this limit, the equation (3.12) is easily integrated to yield a constant of motion

$$\frac{a_3\dot{B}}{a_1 a_2} \equiv L_3 \qquad \text{(constant)}\,. \tag{3.18}$$

With the vanishing potential, B manifests itself by non-vanishing time derivatives. In the dual variable, it corresponds to the homogeneous gradient along x^3-direction. The spacetime evolution with the dilaton rolling in this case was studied in reference [6]. Here we have assumed that the dilaton is stabilized by some mechanism. Following reference [6] we introduce a new time coordinate λ via the relation $d\lambda = L_3 dt/a_1 a_2 a_3$. Then the equations (3.13)–(3.16) can be written as

$$\begin{aligned} \alpha_1'\alpha_2' + \alpha_2'\alpha_3' + \alpha_3'\alpha_1' &= \frac{1}{4}a_1^2 a_2^2\,, && (3.19)\\ \alpha_1'' = \alpha_2'' &= 0\,, && (3.20)\\ \alpha_3'' &= \frac{1}{2}a_1^2 a_2^2\,, && (3.21) \end{aligned}$$

where the prime denotes the differentiation with respect to λ.

The solutions for α_1 and α_2 are trivial

$$\alpha_1 = C_1\lambda\,, \qquad \alpha_2 = C_2\lambda\,, \tag{3.22}$$

where $C_{1,2}$ are constants and we omitted the integration constants corresponding simply to re-scaling of scale factors. The α_3-equation (3.21) is also easily integrated to give

$$\alpha_3 = \frac{e^{2(C_1+C_2)\lambda}}{8(C_1+C_2)^2} + C_3\lambda\,. \tag{3.23}$$

The constraint equation (3.19) relates $C_{1,2}$ and C_3 by $C_3 = -C_1C_2/(C_1+C_2)$. Then the relation between λ and $L_3 t$ is explicitly given by

$$\begin{aligned} L_3 t &= \int^\lambda d\lambda\; a_1(\lambda)a_2(\lambda)a_3(\lambda)\\ &= \int^\lambda d\lambda \exp\left[(C_1+C_2+C_3)\lambda + \frac{e^{2(C_1+C_2)\lambda}}{8(C_1+C_2)^2}\right]\\ &= [8(C_1+C_2)]^{\frac{C_1+C_2+C_3}{C_1+C_2}} \int^x dy\; y^{-P} e^y\,, \end{aligned} \tag{3.24}$$

where $x = e^{2(C_1+C_2)\lambda}/8(C_1+C_2)^2$ and $P = (C_1+C_2-C_3)/2(C_1+C_2)$. The evolution of scale factors for large $L_3 t$ is given by

$$a_{1,2} \propto (\log L_3 t)^{q_{1,2}}\,, \qquad a_3 \propto L_3 t\,, \tag{3.25}$$

where $q_{1,2} = C_{1,2}/2(C_1+C_2)$. Therefore, with non-vanishing $B_{12}(t)$, only a_3 grows significantly and the spatial anisotropy develops. This can be seen clearly by considering the ratio $H_3/H_{1,2}$ where $H_i \equiv \dot{a}_i/a_i$. Taking $C_1 = C_2$ and thus $q_{1,2} = 1/4$, we get

$$\frac{H_3}{H_{1,2}} = 4\log L_3 t \tag{3.26}$$

which grows as time elapses. Finally, we remark that such anisotropy cannot be overcome by some other type of isotropic energy density in an expanding universe. Assume the isotropic universe ($a_i = a$) driven by, e.g, radiation energy density $\rho_{\rm R}$. Then, one finds $\rho_B \propto 1/a^2$ from (3.7) and (3.18) and thus $\rho_B/\rho_{\rm R} \propto a^2$, which implies that the late-time isotropic solution can be realized only in a contracting universe [6].

3.2.2 The B oscillation

Let us now take into consideration the effect of space-filling D-brane. To get sensible solution, we fine-tune the bulk cosmological constant term to cancel the brane tension, that is $\Lambda = -m_B^2/2$, so that the effective four-dimensional cosmological constant vanishes. It is convenient to define a new variable $b \equiv B/a_1 a_2$. Then we can rewrite the full equations as follows

$$\dot\alpha_1^2 + 2\dot\alpha_1\dot\alpha_3 = \frac{1}{4}\left(\dot b + 2\dot\alpha_1 b\right)^2 + \frac{m_B^2}{2}\left(\sqrt{1+b^2}-1\right), \quad (3.27)$$

$$\ddot\alpha_1 + \dot\alpha_1\left(2\dot\alpha_1 + \dot\alpha_3\right) = \frac{m_B^2}{2}\left(\sqrt{1+b^2}-1\right), \quad (3.28)$$

$$\ddot\alpha_3 + \dot\alpha_3\left(2\dot\alpha_1 + \dot\alpha_3\right) = \frac{1}{2}\left(\dot b + 2\dot\alpha_1 b\right)^2 + \frac{m_B^2}{2}\left(\frac{1}{\sqrt{1+b^2}}-1\right), \quad (3.29)$$

$$\ddot b + \left(2\dot\alpha_1 + \dot\alpha_3\right)\dot b + \left(2\ddot\alpha_1 + 2\dot\alpha_1\dot\alpha_3 + \frac{m_B^2}{\sqrt{1+b^2}}\right) b = 0\,. \quad (3.30)$$

With the help of (3.27, 3.30) can be written as

$$\ddot b + \left(2\dot\alpha_1 + \dot\alpha_3\right)\dot b + \left[-4\dot\alpha_1^2 + m_B^2\left(\sqrt{1+b^2} - 1 + \frac{1}{\sqrt{1+b^2}}\right)\right] b = 0\,, \quad (3.31)$$

which is valid only for the B-dominated case, but more convenient for numerical analysis. Here we have put $\kappa_4 \equiv 1$.

First we examine the evolution of $b(t)$ and scale factors qualitatively. From (3.27)–(3.30), we see that the natural time scale is m_B^{-1} and $m_B t$ is the dimensionless time variable. Suppose b starts to roll from an initial value b_0, while the universe is isotropic in the sense that $\dot\alpha_{10} = \dot\alpha_{30}$. We assume initially $a_{10} = a_{30} = 1$ ($\alpha_{10} = \alpha_{30} = 0$) and $\dot B_0 = 0$ so that $b_0 = B_0$ and $\dot b_0 = -2\dot\alpha_{10} B_0$. While b is much larger than unity, the rapid expansion of α_1 due to the large potential proportional to $m_B^2 b$ drives b in feedback to drop very quickly to a small value of order one. Our numerical analysis in figure 1 shows that this happens within $m_B t < 2$ up to reasonably large value of b_0 for which the numerical solution is working. The behavior of $b(t)$ after this point is almost universal irrespective of the initial value b_0 if it is much larger than unity.

Once b becomes smaller than unity, the quadratic term of mass dominates over the expansion and b begins to oscillate about $b = 0$. Then the expansion of the universe provides the slow decrease of the oscillation amplitude. The situation is the same as that of the coherently oscillating scalar field such as the axion or the moduli in the expanding universe. For small b, the energy-momentum tensor of the oscillating B field is given by $T_\mu{}^\nu = \mathrm{diag}[-\rho, p_1, p_2, p_3]$ where

$$\rho = \frac{1}{4}\left(\dot b + 2\dot\alpha_1 b\right)^2 + \frac{1}{2}m_B^2\left(\sqrt{1+b^2} - 1\right) \approx \frac{1}{4}\left(\dot b^2 + m_B^2 b^2\right), \quad (3.32)$$

$$p_1 = p_2 = -\frac{1}{4}\left(\dot b + 2\dot\alpha_1 b\right)^2 - \frac{1}{2}m_B^2\left(\frac{1}{\sqrt{1+b^2}} - 1\right) \approx -\frac{1}{4}\left(\dot b^2 - m_B^2 b^2\right), \quad (3.33)$$

$$p_3 = \frac{1}{4}\left(\dot b + 2\dot\alpha_1 b\right)^2 - \frac{1}{2}m_B^2\left(\sqrt{1+b^2} - 1\right) \approx \frac{1}{4}\left(\dot b^2 - m_B^2 b^2\right). \quad (3.34)$$

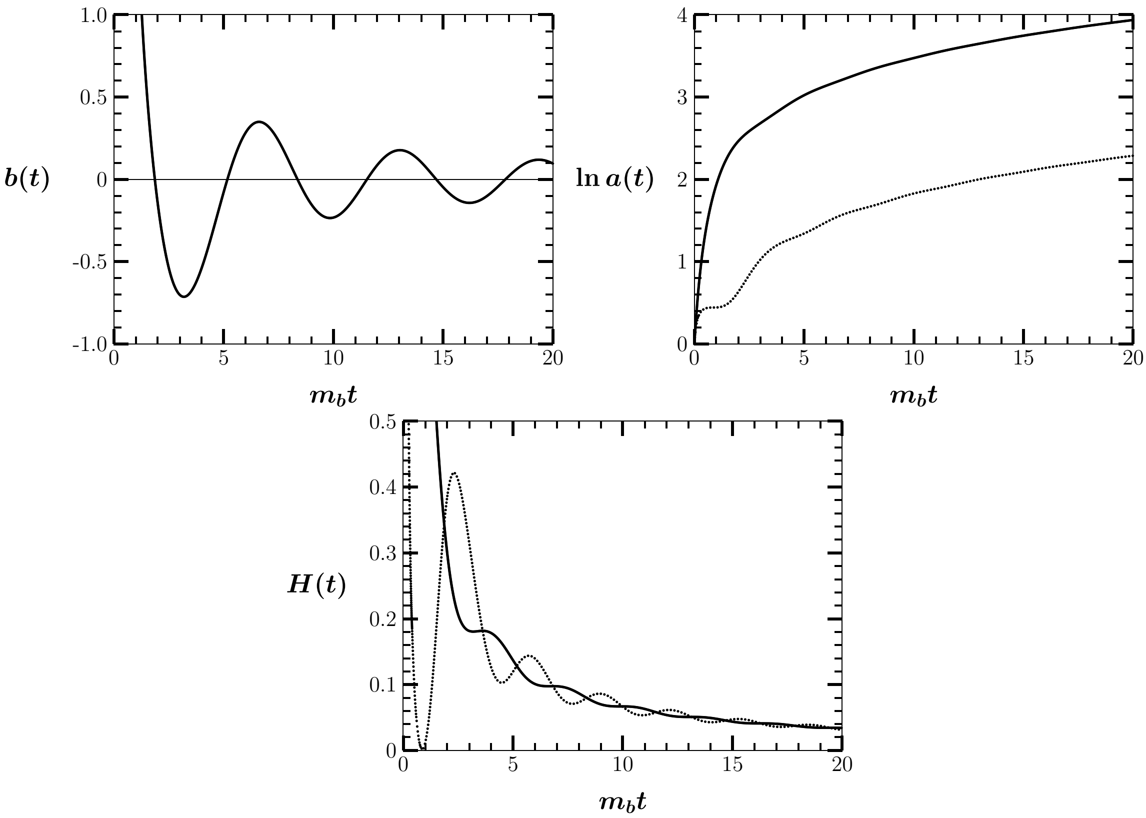

Figure 1: Numerical solutions for $b_0 = 100$. *(a)* The evolution of $b(t)$. *(b)* Scale factors $a_1(t)$ (solid line) and $a_3(t)$ (dotted line). *(c)* Expansion rates $H_1(t)$ (solid line) and $H_3(t)$ (dotted line).

With the expansion of the universe neglected, the equation of motion for b is then approximated by

$$\ddot{b} + m_B^2 b \approx 0\,. \tag{3.35}$$

Since the oscillation is much faster than the expansion, we can use the time-averaged quantities over one period of oscillation for the evolution of spacetime. The equation (3.35) gives the relation $\langle \dot{b}^2 \rangle = \langle m_B^2 b^2 \rangle$. Thus, the oscillating B field has the property $p_1, p_2, p_3 \approx 0$ and behaves like homogeneous and isotropic matter. This justifies the name of *B-matter*. Therefore, after b begins to oscillate, the isotropy of the universe is recovered.

To quantify how the universe recovers isotropy, let us define the parameter

$$s \equiv \sqrt{2}\frac{H_1 - H_3}{2H_1 + H_3}\,.$$

When $H_1 \approx H_3$, the evolution of the quantity $s(t)$ is determined by

$$\dot{s} = \frac{1}{6}\left(\frac{4p_1 - p_3 - 9H^2}{H}\right) s + \frac{1}{3}\frac{p_1 - p_3}{H} + \mathcal{O}(s^2)\,, \tag{3.36}$$

where $H \equiv \sum_i H_i/3$. At late time, the equation (3.36) is approximated as $\dot{s} \approx -(3/2)Hs$

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and thus one finds $s \propto 1/t$. Since H is also proportional to $1/t$, the initial and final anisotropy $s_{i,f}$ follow the relation

$$s_f = s_i \frac{H_f}{H_i}\,, \tag{3.37}$$

where $H_{i,f}$ denotes the initial and final Hubble parameter, respectively. To get an idea of how fast the anisotropy disappears, let us consider $H_f \sim 10^{-15}$ GeV which corresponds to the Hubble parameter at the electroweak symmetry breaking scale of temperature, $T \sim 100\,\text{GeV}$. Taking the initial condition $s_i \sim 1$ around the beginning of the B oscillation $H_i \sim m_B$, we find that the final anisotropy can be completely neglected for reasonable value of m_B.

For $m_B t \gg 1$ with $H_1 \simeq H_2 \simeq H_3$ and $b \ll 1$, the asymptotic solution of (3.27)–(3.30) can be explicitly found to yield

$$H \propto \frac{2}{3t}\,, \qquad b \propto \frac{1}{t}\,, \qquad \rho_{B,b} \propto \frac{1}{t^2} \tag{3.38}$$

which shows the usual matter-like evolution. That is, the total energy density of the B-matter diminishes like $\rho \propto 1/a^3$ even though the amplitude $B(t)$ itself grows like $B(t) \propto a^{1/2}$.

Our qualitative results can be confirmed by solving the equations (3.28), (3.29), and (3.31) numerically. Figure 1 shows the numerical solutions for the initial value $b_0 = 100$. For the large value of b_0, a_1 grows very fast while a_3 is frozen until b becomes smaller than unity. Then b begins to oscillate and the universe becomes isotropic again in that the expansion rates, $\dot{\alpha}_1$ and $\dot{\alpha}_3$, converge and finally the universe becomes B-matter dominated.

3.3 Evolution of $B(t)$ in the RW background

When the universe is dominated by homogeneous and isotropic matter or radiation, while the B field contributes a minor fraction, the background geometry is described by the Robertson-Walker metric and the equation for $b(t)$ is

$$\ddot{b} + 3\frac{\dot{a}}{a}\dot{b} + \left(2\frac{\ddot{a}}{a} + \frac{m_B^2}{\sqrt{1+b^2}}\right) b = 0\,. \tag{3.39}$$

In this subsection, we study the evolution of $b(t)$ during inflation and the radiation dominated era.

3.3.1 Evolution during inflation

During inflation, $\dot{a}/a \approx H$ (constant) and $\ddot{a}/a \approx H^2$. We are assuming that the inflaton dominates over the antisymmetric tensor field, which means $H^2 \gg m_B^2(\sqrt{1+b^2} - 1)$. Defining a dimensionless time variable as Ht, we write the equation (3.39) as

$$\ddot{b} + 3\dot{b} + \left(2 + \frac{m_B^2/H^2}{\sqrt{1+b^2}}\right) b = 0\,. \tag{3.40}$$

Due to $2H^2$ term, dominating over the oscillatory m_B^2 term, b exponentially dies away. Neglecting m_B^2 term, we have the solution $b \approx b_0 e^{-2Ht}$, corresponding to a frozen B field at some value B_0, while its energy density decreases exponentially during inflation. Therefore inflation dilutes away the pre-existing B field.

3.3.2 Evolution during radiation dominated era

During the radiation dominated era, $\dot{a}/a = 1/2t$ and $\ddot{a}/a = -1/4t^2$. Defining the dimensionless time variable as $m_B t$, we rewrite the equation (3.39) as

$$\ddot{b} + \frac{3}{2t}\dot{b} + \left(-\frac{1}{2(m_B t)^2} + \frac{1}{\sqrt{1+b^2}}\right) b = 0\,. \tag{3.41}$$

As in the case of B domination, the amplitude of b shortly diminishes and becomes smaller than unity. Then the equation (3.41) is approximated as a linear equation in b to yield the usual damped-oscillation;

$$b(t) \approx \sqrt{\frac{2}{\pi}}\frac{1}{(m_B t)^{3/4}}\left[C_1 \cos\left(m_B t + \frac{3\pi}{8}\right) + C_2 \sin\left(m_B t + \frac{3\pi}{8}\right)\right] \tag{3.42}$$

for $m_B t \gg 1$. In the regime, the energy density of the B-matter is found to be

$$\rho \propto \frac{1}{(m_B t)^{3/2}} \propto 1/a^3\,. \tag{3.43}$$

That is, for $m_B t \gg 1$, the oscillation dominates over the expansion and the oscillating B field behaves like the usual matter during the radiation dominated era. As a consequence the radiation will be overtaken by the B-matter eventually and the cosmological evolution follows then the result of the subsection 3.2.

4. Conclusion

We have investigated the cosmology with the antisymmetric tensor field in a D-brane universe. A peculiar feature of this system is that the antisymmetric tensor field becomes massive due to its coupling to the massless U(1) gauge field on the brane and thus its cosmological consequences are drastically different from those of the massless antisymmetric tensor field. Analyzing the Bianchi type I cosmology describing the simplest form of an anisotropic universe, we found time-dependent homogeneous solutions exhibiting the matter-like (pressureless) behavior of the massive antisymmetric tensor field (B-matter). For instance, the B-matter amplitude oscillating with the frequency of m_B gets damped due to expansion. The energy density scales like $\rho \propto 1/a^3$ and the averaged pressure becomes vanishingly small. As a consequence, the initial anisotropy dies away rapidly and the isotropic universe is recovered at late time.

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